Detection Basics

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CMPE591
Outline

1 Recall
   - Optimality Frameworks

2 Example Questions
   - Bayesian Optimality
   - Minimax Optimality
   - NP Optimality
Goal

Main goal is to decide between two hypotheses

\[ H_0 \quad Y \sim p_0 \]
\[ H_1 \quad Y \sim p_1 \]
**Optimality Frameworks**

**Bayesian Optimality (again)**

<table>
<thead>
<tr>
<th>Known Parameters</th>
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<tbody>
<tr>
<td>( \pi_0, \pi_1 )</td>
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<tr>
<td>( c_{ij} )</td>
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<tr>
<td>( p_i )</td>
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**Goal**

Minimizing the average cost.
MiniMax Optimality (again)

**Known Parameters**
- $c_{ij}$
- $p_i$

**Goal**
Minimizing maximum cost or equivalently finding the equalizer rule.

$$\min_{\delta} \max \{ R_0(\delta), R_1(\delta) \}$$
Neyman-Pearson Optimality (again)

Known Parameters

- $p_i$

Goal

Maximizing $P_D = \int_{\Gamma_1} p_1(y) \, dy$ \quad s.t. $P_F = \int_{\Gamma_1} p_0(y) \, dy \leq \alpha$

Remark

Maximum $P_D$ is achieved when $P_F = \alpha$ for continuous conditional pdfs.
Example

Suppose $Y$ is a random variable that, under $H_0$, has pdf

$$p_0(y) = \begin{cases} \frac{2}{3}(y + 1) & 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

and, under hypothesis $H_1$, has pdf

$$p_1(y) = \begin{cases} 1 & 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the Bayes rule and minimum Bayes risk for testing $H_0$ versus $H_1$ with uniform costs and equal priors.
Example

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Find the Minimax rule and Minimax risk for uniform costs.
Example

Suppose $Y$ is a random variable that, under $H_0$, has pdf

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and, under hypothesis $H_1$, has pdf

$$p_1(y) = \begin{cases} 1 & 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the Neyman-Pearson rule and the corresponding detection probability for false-alarm probability $\alpha \in (0, 1)$. 